DADE GROUP

12.4.7 — 12.4.15

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THE POINT: INTRODUCE A CONSTRUCTION THAT CAN BE USED TO RECOVER A LOT OF THE OPERATIONS WE CAN DO ON $\mathrm{Perm}_K(P)$ LIKE Res, Iso, ...

12.4.7 GENERALIZED TENSOR INDUCTION

LET P AND Q BE FINITE p-GROUPS AND U BE A FINITE (Q,P)-BISET.

IF X IS A FINITE P-SET, DENOTE BY

$$T_U(X) := \mathrm{Hom}_P(U^{op}, X) = \{ \varphi \mid \varphi : U \to X \text{ s.t. } \underset{\substack{\| \\ \varphi(g \cdot_{op} u)}}{\varphi(ug^{-1})} = g\varphi(u), \forall g \in P, \forall u \in U \}$$

NOTE: $T_U(X) = \mathrm{Hom}_P(U^{op}, X)$ IS A FINITE Q-SET WITH THE FOLLOWING ACTION:

GIVEN $\varphi \in T_U(X)$, $h \in Q$ $\boxed{(h\cdot\varphi)(u) = \varphi(h^{-1}u)}$ $\forall u \in U$.

NEED TO CHECK $(h\cdot\varphi) \in \mathrm{Hom}_P(U^{op}, X)$?

NOTE $(h\cdot\varphi)(u\cdot g^{-1}) \overset{\text{by } Q}{=} \varphi(h^{-1}ug^{-1}) \overset{\text{by } P}{=} g\cdot(\varphi(h^{-1}u))$

$\underset{\text{by } Q}{=} g\cdot((h\cdot\varphi)(u))$ so $(h\cdot\varphi) \in T_U(X)$, $\forall g \in P$.

EXAMPLE:

LET $Q \leq P$, AND LET $U = P$ as (Q,P)-biset, X P-SET

NOTE $T_U(X) = \mathrm{Hom}_P(P, X) \cong \mathrm{Res}^P_Q X$ } on objects

$\alpha \longmapsto \alpha(1) \in X$

$$(h \cdot \alpha)(1) \underset{\text{by } Q}{=} \alpha(h^{-1}) \underset{\text{by } P}{=} h \cdot (\alpha(1))$$

NOTE T_U GIVE US A MAP:

$$T_U : \mathrm{Obj}(\underline{\mathrm{Perm}}_K(P)) \longrightarrow \mathrm{Obj}(\underline{\mathrm{Perm}}_K(Q))$$

$$X \longmapsto T_U(X)$$

CAN WE HAVE A COMPATIBLE MAP ON ~~M~~ MORPHISM?

LET $m \in \mathrm{Hom}_{\underline{\mathrm{Perm}}_K(P)}(X, Y)$ SO RECALL

$m : Y \times X \to K$ s.t. $m(gy, gx) = m(y, x)$ for all $g \in P$

$\forall (y, x) \in Y \times X$

WANT $T_U(m) \in \mathrm{Hom}_{\underline{PERm}_K(Q)}\big(T_U(X), T_U(Y)\big)$

DEFINE:

$$T_U(m) : T_U(Y) \times T_U(X) \longrightarrow K$$

$$(\varphi, \psi) \longmapsto T_U(m)(\varphi, \psi) = \prod_{u \in [U/P]} m\Big(\overset{\in X}{\varphi(u)}, \overset{\in Y}{\psi(u)}\Big)$$

WHERE $[U/P]$ IS A SET OF REPRESENTATIVES OF U/P.

WELL DEFINED? $m(\varphi(ug^{-1}), \psi(ug^{-1})) = m(g\,\varphi(u), g\,\psi(u))$

WITH $g \in P$. $\qquad = m(\varphi(u), \psi(u))$

AS m, φ, ψ ARE P-EQUIVARIANT.

12.4.8. Lemma : *This definition yields a morphism $T_U(m)$ from $T_U(X)$ to $T_U(Y)$ in the category $\underline{\mathsf{Perm}}_k(Q)$.*

PROOF: NEED TO SHOW $T_U(m)$ IS Q-INVARIANT, $h \in Q$

$$T_U(m)(h^{-1}\varphi, h^{-1}\varphi) \overset{1}{=} \prod_{u \in [U/P]} m(h^{-1}\varphi(u), h^{-1}\varphi(u)) \quad \text{by DEF } T_U(m)$$

by DEF OF Q ACTION

$$\overset{2}{=} \prod_{u \in [U/P]} m(\varphi(h.u), \varphi(hu))$$

THE IMAGE OF all $h.u$ on $[U/P]$

IS ANOTHER SET OF REPRESENTATIVES.

$$= \prod_{u' \in [V/\rho]} m(\varphi(u'), \psi(u')) = T_V(m)(\varphi, \psi)$$

SO THE MATRIX $T_V(m)$ IS Q- INVARIANT.

SO $T_V(m)$ IS A MORPHISM IN $\underline{\mathrm{Perm}}_K(Q)$. $\square$

THUS ONE CAN ASK IS $T_V(m)$ A FUNCTOR?

12.4.9. Lemma : [19, Lemma 2.2] *Let k be a field of characteristic p. Let P and Q be finite p-groups, and let U be a finite (Q,P)-biset. Then T_U is a functor from $\underline{\mathsf{Perm}}_k(P)$ to $\underline{\mathsf{Perm}}_k(Q)$.*

PROOF: LEFT TO SHOW: ① $T_U(Id_X) = Id_{T_U(X)}$

② $T_U(m \cdot n) = T_U(m) \circ T_U(n)$

① LET X BE FINITE P-SET, $Id_X = \delta_X(y,y') = \begin{cases} 1, & y = y' \\ 0, & y \neq y' \end{cases}$

LET $\varphi, \psi \in T_U(X)$

$$T_U(\delta_X)(\varphi,\psi) = \prod_{u \in U/P} \delta_X(\varphi(u), \psi(u)) = \begin{cases} 0, & \varphi \neq \psi \\ 1, & \varphi = \psi \end{cases}$$

SO ① ✓. AS φ, ψ ARE P-EQUIVARIANT

② ... LEFT TO THE READER...

NEED TO SHOW $m \mapsto T_U(m)$ IS MULTIPLICATIVE

$$T_U(m \cdot n)(\varphi, \theta) = T_U(m) \circ T_U(n)(\varphi, \theta)$$

WHERE $n : Z \to X$ AND $m : X \to Y$

$\theta \in T_U(Z)$, $\varphi \in T_U(Y)$

NOTE: UNDER CERTAIN CONDITIONS THERE ARE MAPS FROM $F : \mathrm{Perm}_K(P) \to \mathrm{Perm}_K(Q)$. SO IT IS NATURAL TO ASK IS THERE A U A FINITE (Q,P)-BISET MAKES THE DIAGRAM COMMUTES OR THE OTHER WAY AROUND

$$
\begin{tikzcd}
\mathrm{Perm}_K(P) \arrow[r, "e_P"] \arrow[d, "T_U"'] & \mathrm{Perm}_K(P) \arrow[d, "F"] \\
\mathrm{Perm}_K(Q) \arrow[r, "e_Q"] & \mathrm{PERm}_K(Q)
\end{tikzcd}
$$

12.4.10 EXAMPLE: ELEMENTARY BISETS:

EXAMPLE ①: F = RESTRICTION

LET $Q \leq P$, AND LET $U = P$ as (Q,P)-BISET, X P-SET

NOTE $T_U(X) = \mathrm{Hom}_P(P, X) \cong \mathrm{Res}^P_Q X$
$\alpha \longmapsto \alpha(1) \in X$ } on objects

IF $m: X \to Y$ IN $\mathrm{PERM}_K(P)$ THEN } ON MORPHISM

$$T_U(m)(\varphi, \psi) = \prod_{u \in P/P} m(\varphi(u), \psi(u)) = m(\varphi(1), \psi(1))$$

So $T_U(m) = m$

SO T_U IS JUST THE RESCTRICTION FUNCTOR.

$$\begin{tikzcd} \underline{\mathrm{Perm}}_K(P) \arrow[r, hook, "e_P"] \arrow[d, "T_U"'] & \mathrm{Perm}_K(P) \arrow[d, "\mathrm{Res}^P_Q"] \\ \underline{\mathrm{Perm}}_K(Q) \arrow[r, hook, "e_Q"] & \mathrm{Perm}_K(Q) \end{tikzcd}$$

EXAMPLE 2: TENSOR INDUCTION:

LET $Q \leq P$, AND LET $V = P$ as (P, Q)-biset, X Q-set

$$T_V(X) = \mathrm{Hom}_Q(V^{op}, X) = \mathrm{Hom}_Q(P, X) \cong \prod_{s \in P/Q} X$$

$$\left(\begin{array}{l} \varphi \longmapsto \prod_{s \in P/Q} (\varphi(s)) \\ \varphi \longleftarrow \prod_{s \in P/Q} x_s \end{array} \right)$$

by $\varphi(sh) = h^{-1} x_s$, $h \in Q$

NOTE: UNDER THE ISOMORPHISM $\mathrm{Hom}_Q(P, X) \cong \prod_{s \in P/Q} X$

ONE CAN GIVE AN ACTION OF $g \in P$ ON $\prod_{s \in P/Q} X$

ONE CAN CHECK: $g \cdot (x_s)_{s \in S} = \left(h_{\varepsilon^{-1}(s)} x_{\varepsilon^{-1}(s)} \right)_{s \in P/Q}$

by $g \cdot s = \varepsilon(s) h_s$ WHERE ε IS A PERMUTATION OF P/Q.

$h_s \in Q$.

ON ANOTHER NOTE; GIVEN K ~~X~~ ~~A~~ WE CAN DEFINE

$$\mathrm{Ten}_Q^P(KX) = \bigotimes_{s \in P/Q} s \otimes KX$$ CALLED THE TENSOR INDUCED MODULE

AS BEFORE, $g\left(\bigotimes_{s \in P/Q} s \otimes m_s\right) = \bigotimes_{s \in P/Q}\left(s \otimes h_{\epsilon^{-1}(s)}\, m_{\epsilon^{-1}(s)}\right)$

WHERE $g \in P$,

SO NOTE $K\,T_V(X) \cong K \prod_{s \in P/Q} X \cong \mathrm{Ten}_Q^P(KX)$

ONE CAN CHECK!

$$\begin{tikzcd}
\underline{\mathsf{Perm}}_k(Q) \arrow[r, "e_Q"] \arrow[d, "T_V"'] & \mathsf{Perm}_k(Q) \arrow[d, "\mathrm{Ten}_Q^P"] \\
\underline{\mathsf{Perm}}_k(P) \arrow[r, "e_P"] & \mathsf{Perm}_k(P)
\end{tikzcd}$$

EXAMPLE 3: INFLATION:

LET $N \trianglelefteq P$, $U = P/N$ as $(P, P/N)$-BISET

LET X BE A P/N-SET

$$T_U(X) = \mathrm{Hom}_{P/N}(P/N, X) \cong \mathrm{Inf}_{P/N}^{P} X$$

$$\varphi \mapsto \varphi(1)$$

NOTE $T_U(m) = m$ AS $U/_{P/N} = P/N / P/N = 1$.

(SAME ARGUMENT WITH Res)

SO T_U IS JUST THE INFLATION FUNCTOR

$$
\begin{tikzcd}
\underline{\mathsf{Perm}}_k(P/N) \arrow[r, "e_{P/N}"] \arrow[d, "T_U"'] & \mathsf{Perm}_k(P/N) \arrow[d, "\mathrm{Inf}^P_{P/N}"] \\
\underline{\mathsf{Perm}}_k(P) \arrow[r, "e_P"] & \mathsf{Perm}_k(P)
\end{tikzcd}
$$

EXAMPLE 4: BRAUER QUOTIENT:

LET $V = P/N$ as $(P/N, P)$-BISET THEN WE HAVE

$$T_V(X) \cong X^N \quad \text{AND} \quad kX^N \cong (kX)[N] \quad \text{SO} (\dots)$$

$$\|$$

$$\mathrm{Hom}_P(P/N, X)$$

$$
\begin{tikzcd}
\underline{\mathsf{Perm}}_k(P) \arrow[r, "e_P"] \arrow[d, "T_V"'] & \mathsf{Perm}_k(P) \arrow[d, "\mathrm{Br}^P_N"] \\
\underline{\mathsf{Perm}}_k(P/N) \arrow[r, "e_{P/N}"] & \mathsf{Perm}_k(P/N)
\end{tikzcd}
$$

EXAMPLE 5: ISOMORPHISM:

LET $f: P \xrightarrow{\sim} Q$, X BE A FINITE P-SET

$U = P$ as (Q,P)-BISET

$T_U(X) \cong \mathrm{Iso}_P^Q(X)$ SO ONE CAN SHOW:

$$\begin{tikzcd}
\underline{\mathrm{Perm}}_K(P) \arrow[r, "e_P"] \arrow[d, "T_U"'] & \mathrm{Perm}_K(P) \arrow[d, "\mathrm{Iso}_P^Q"] \\
\underline{\mathrm{Perm}}_K(Q) \arrow[r, "e_Q"] & \mathrm{Perm}_K(Q)
\end{tikzcd}$$

SO T_U IS JUST ISO.

NOTE: RESTRICTION, TENSOR INDUCTION, INFLATION, BRAUER QUOTIENT, ISOMORPHISM CAN BE RECOVERED by MEANS OF THIS T_U. HENCE WE HAVE THE FOLLOWING DEFINITION

BECAUSE OF TENSOR INDUCTION!

DEFINITION: THE FUNCTOR T_U IS CALLED THE GENERALIZED TENSOR INDUCTION ASSOCIATED TO U.

SOME PROPERTIES OF T_U!

12.4.12. Proposition : *Let k be a field of characteristic p, let P and Q be finite p-groups, and let U be a finite (Q,P)-biset.*

1. *If X is a one element P-set, then $T_U(X)$ is a one element Q-set.*
2. *If X and Y are finite P-sets, there is an isomorphism*

$$T_U(X\times Y)\cong T_U(X)\times T_U(Y)\ ,$$

which is functorial in X and Y: there is a commutative diagram of categories and functors

$$\begin{tikzcd}
\underline{\mathsf{Perm}}_k(P)\times\underline{\mathsf{Perm}}_k(P) \arrow[r, "\times"] \arrow[d, "T_u\times T_U"'] & \underline{\mathsf{Perm}}_k(P) \arrow[d, "T_U"] \\
\underline{\mathsf{Perm}}_k(Q)\times\underline{\mathsf{Perm}}_k(Q) \arrow[r, "\times"] & \underline{\mathsf{Perm}}_k(Q)\ .
\end{tikzcd}$$

3. *If X and Y are finite P-sets, let $\sigma : X\times Y\to Y\times X$ denote the switch morphism. Then up to the isomorphism of Assertion 2, the morphism $T_U(\sigma)$ is the switch morphism $T_U(X)\times T_U(Y)\to T_U(Y)\times T_U(X)$.*
4. *If U' is another finite (Q,P)-biset, then the functors $T_{U\sqcup U'}$ and $T_U\times T_{U'}$ are isomorphic.*

PROOF: (1) IF $|X| = 1$ THEN THERE IS A UNIQUE MORPHISM OF P-SETS FROM ANY P-SET TO X.

(2) 2. *If X and Y are finite P-sets, there is an isomorphism*

$$T_U(X \times Y) \cong T_U(X) \times T_U(Y)\,,$$

THERE IS A CANONICAL ISOMORPHISM OF Q-SETS

$$\begin{array}{ccccc} T_U(X \times Y) = \mathrm{Hom}_P(U^{op}, X \times Y) \cong & \mathrm{Hom}_P(U^{op}, X) & \times & \mathrm{Hom}_P(U^{op}, Y) \\ & \| & & \| \\ & T_U(X) & \times & T_U(Y) \end{array}$$

ON MORPHISM: GIVEN $m : X \to X'$ AND $n : Y \to Y'$

WE DEFINE $m \times n : X \times Y \to X' \times Y'$ i.e

$$(m \times n)(x', y', x, y) = m(x', x)\, n(y', y).$$

ALL WE NEED TO SHOW IS:

$$T_U(m \times n) : T_U(X) \times T_U(Y) \to T_U(X') \times T_U(Y')$$

$$\|$$

$$T_U(m) \times T_U(n) : T_U(X) \times T_U(Y) \to T_U(X') \times T_U(Y')$$

by SHOWING: $T_U(m \times n)(\varphi', \varphi) = T_U(m)(\varphi'_{X'}, \varphi_X) \cdot T_U(n)(\varphi'_{Y'}, \varphi_Y)$

$\varphi' : U^{op} \to X' \times Y'$, $\varphi : U^{op} \to X \times Y$

③

3. *If X and Y are finite P-sets, let $\sigma : X \times Y \to Y \times X$ denote the switch morphism. Then up to the isomorphism of Assertion 2, the morphism $T_U(\sigma)$ is the switch morphism $T_U(X) \times T_U(Y) \to T_U(Y) \times T_U(X)$.*

THE SWITCH MORPHISM $\sigma : X \times Y \to Y \times X$ IS DEFINED by

$$\sigma(y, x, x', y') = \delta_{x,x'}\, \delta_{y,y'}$$

THEN $T_U(\sigma) : \big(T_U(Y) \times T_U(X)\big) \times \big(T_U(X) \times T_U(Y)\big) \to K$

$$T_U(\sigma)(\varphi, \psi, \psi', \varphi') = \prod_{u \in U/P} \sigma(\varphi(u), \psi(u), \psi'(u), \varphi'(u))$$

AS $\varphi, \psi, \psi', \varphi'$ ARE P-EQUIVARIANT, FOR $T_U(\sigma)$ TO BE NON-ZERO NEED $\varphi = \varphi'$ AND $\psi = \psi'$.

SO $T_U(\sigma)$ IS THE SWITCH MORPHISM.

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4. If U' is another finite (Q, P)-biset, then the functors $T_{U \sqcup U'}$ and $T_U \times T_{U'}$ are isomorphic.

ON OBJECTS: LET X BE A P-SET

$$T_{U \sqcup U'}(X) = \mathrm{Hom}_P((U \sqcup U')^{op}, X) \cong \underbrace{\mathrm{Hom}_P(U^{op}, X)}_{= T_U(X)} \times \underbrace{\mathrm{Hom}_P(U'^{op}, X)}_{= T_{U'}(X)}$$

GIVEN BY $\left(\varphi : (U \sqcup U')^{op} \to X\right) \longmapsto \left(\varphi|_U, \varphi|_{U'}\right)$

ON MORPHISM: LET $m : Y \times X \to K$ BE A MORPHISM FROM X TO Y. WANT TO SHOW $T_{U \sqcup U'}(m) = T_U(m) \times T_{U'}(m)$

$$T_{U \sqcup U'}(m)(\varphi, \psi) = \prod_{u \in [U \sqcup U'/\rho]} m(\varphi(u), \psi(u)) \quad \text{by DEF } T$$

$$\text{by } \sqcup. \qquad = \prod_{u \in [U/\rho]} m(\varphi(u), \psi(u)) \cdot \prod_{u' \in [U'/\rho]} m(\varphi(u') \psi(u'))$$

$$\text{by } T \qquad = T_U(m)\left(\varphi|_U, \psi_U\right) \cdot T_{U'}(m)\left(\varphi|_{U'}, \psi|_{U'}\right)$$

SO $T_{U \sqcup U'}(m) = T_U(m) \times T_{U'}(m)$ $\square$

RECAP: SOME PROPERTIES OF T_U!

12.4.12. Proposition : *Let k be a field of characteristic p, let P and Q be finite p-groups, and let U be a finite (Q,P)-biset.*

1. *If X is a one element P-set, then $T_U(X)$ is a one element Q-set.*
2. *If X and Y are finite P-sets, there is an isomorphism*

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\underline{\mathsf{Perm}}_k(Q) \times \underline{\mathsf{Perm}}_k(Q) \arrow[r, "\times"] & \underline{\mathsf{Perm}}_k(Q)\ .
\end{tikzcd}$$

3. *If X and Y are finite P-sets, let $\sigma : X \times Y \to Y \times X$ denote the switch morphism. Then up to the isomorphism of Assertion 2, the morphism $T_U(\sigma)$ is the switch morphism $T_U(X) \times T_U(Y) \to T_U(Y) \times T_U(X)$.*
4. *If U' is another finite (Q,P)-biset, then the functors $T_{U \sqcup U'}$ and $T_U \times T_{U'}$ are isomorphic.*

12.4.13 COMPOSITION AND GALOIS TWISTS!

HOW DO WE COMPOSE THOSE OPERATIONS?

LET P, Q, AND R BE FINITE p-GROUPS,

LET U BE A FINITE (Q,P)-BISET AND

V BE A FINITE (R,Q)-BISET.

$V \times_Q U$ BE A FINITE (R,P)-BISET.

$T_U : \underline{\mathrm{Perm}}_K(P) \longrightarrow \underline{\mathrm{Perm}}_K(Q)$ $\quad$ $T_V : \underline{\mathrm{Perm}}_K(Q) \longrightarrow \underline{\mathrm{Perm}}_K(R)$

SO WE CAN FORM: $T_V \circ T_U : \underline{\mathrm{Perm}}_K(P) \longrightarrow \underline{\mathrm{Perm}}_K(R)$

AND ALSO $\quad T_{V \times_Q U} : \underline{\mathrm{Perm}}_K(P) \longrightarrow \underline{\mathrm{Perm}}_K(R)$

ARE $T_V \circ T_U$ AND $T_{V \times_{\mathbb{Q}} U}$ ISOMORPHIC?

SPOILER ALERT! NO

NEED GALOIS TWISTS IN ORDER TO UNDERSTAND THE CONNECTION BETWEEN $T_V \circ T_U$ AND $T_{V \times_{\mathbb{Q}} U}$.

12.4.14 DEFINITION: LET a be AN ENDOMORPHISM OF THE FIELD K. LET P be A FINITE p-GROUP

$$\gamma_{a,P}\ (\gamma_a): \underline{\mathrm{Perm}}_K(P) \to \underline{\mathrm{Perm}}_K(P)$$

OBJECTS: $X \mapsto X$

MORPHISM: $m \mapsto a(m)$ $\quad | \; m: Y \times X \mapsto K$

S.T $a(m)(y,x) = a(m(y,x))$.

WHY IS γ_α A FUNCTOR?

① $\alpha(Id_X) = \alpha(\delta_X) = \delta_X$

② $\alpha(m) \circ \alpha(n) = \alpha(m \circ n)$

FOLLOW AS α IS AN ENDOMORPHISM OF FIELD K.

—//—

FOR $m \in \mathbb{N}$, LET $\gamma(p^m)$ DENOTE THE FUNCTOR γ_α FOR THE ENDOMORPHISM $\alpha : x \mapsto x^{p^m}$ OF K.

THE FUNCTOR $\gamma(p)$ IS CALLED THE FROBENIUS TWIST FUNCTOR

SOME PROPERTIES OF THE γ_a FUNCTOR:

12.4.15. Lemma :

1. *Let a and b be endomorphisms of k. Then for any finite p-groups P*

$$\gamma_{a,P} \circ \gamma_{b,P} \cong \gamma_{a \circ b,P} \ .$$

2. *Let P and Q be finite p-groups, and U be a finite (Q, P)-biset. Then*

$$T_U \circ \gamma_{a,P} = \gamma_{a,Q} \circ T_U \ .$$

PROOF: 1 ON OBJECTS IT IS JUST THE IDENTITY MAP P

ON MORPHISM FOLLOWS BY COMPOSITION OF ENDOMORPHISM

$$\gamma_{a,P} \circ \gamma_{b,P}(m) = a(\gamma_{b,P}(m)) = a(b(m)) = (a \circ b)(m)$$

(2)

2. *Let P and Q be finite p-groups, and U be a finite (Q,P)-biset. Then*

$$T_U \circ \gamma_{a,P} = \gamma_{a,Q} \circ T_U \; .$$

ON OBJECTS: $T_U \circ \gamma_{a,P}(X) = T_U(X) = \gamma_{a,Q} \circ T_U(X)$

γ IS JUST IDENTITY ON OBJECTS

ON MORPHISM: $m : Y \times X \to K$

$$\left(T_U \circ \gamma_{a,P}(m)\right)(\varphi, \psi) = \prod_{u \in U/P} a(m)\left(\varphi(u), \psi(u)\right) \quad \text{by DEF. } T$$

$$= \prod_{u \in U/P} a\left(m[\varphi(u), \psi(u)]\right) \quad \text{by DEF } a$$

$$= a\left(\prod_{\mathfrak{a}\in U/P} m\left(\varphi(\mathfrak{a}),\psi(\mathfrak{a})\right)\right)$$

$$= a\left(T_U(m)(\varphi,\psi)\right) \qquad \text{by DEF. } T$$

$$= \left(\gamma_{a,Q}\circ T_U(m)\right)(\varphi,\psi) \qquad \text{by DEF. } a$$

$\square$

SUMMARY

- DEFINED T_V, FUNCTOR
- SAW SOME Res, Iso, Ten CAN BE RECOVERED BY T_V
- SAW SOME PROPERTIES OF T_V
- ARE $T_V \circ T_U$ AND $T_{V \times_Q U}$ ISOMORPHIC?
- No, need NEW FUNCTOR $\mathcal{J}_a$
- SAW SOME PROPERTIES OF $\mathcal{J}_a$