Mathematics in Life Sciences **Winter Semester 2024**

List of Exercises 6

1. Calculate the following limits. Use L'Hopital's rule if necessary.

$\lim_{x\to 0} \frac{\sin^2(x)}{x^2}$	$\lim_{x\to 0} \frac{e^{2x}-\cos(x)}{x^2}$	$\lim_{x\to\infty} \frac{x}{\ln(x+1)}$
$\lim_{x\to 1} \frac{e^{2x}-e^{x}-e(e-1)}{x^2-1}$	$\lim_{x\to 0} \frac{\cos(\sin(x))-1}{x^2}$	$\lim_{x\to\infty} \frac{\sqrt{4x^2+3}}{2x+1}$
$\lim_{x\to\infty} \frac{\ln(x^2+1)}{\ln(x)}$	$\lim_{x\to\infty} \frac{\sqrt{x^2+3}}{x+2}$	$\lim_{x\to 0} \frac{e^{3x}-1}{2x}$
$\lim_{x\to\infty} \frac{\cos(x)}{\ln(x)}$	$\lim_{x\to 1} \frac{(x-1)\tan(\pi x)}{\sin(\pi x)}$	$\lim_{x\to\infty} \frac{\ln(\ln(x))}{\ln(x)}$
$\lim_{x\to 0^+} \frac{\ln x}{\sin x}$	$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-1}$	$\lim_{x\to 0} \frac{3^x-2^x}{x^2}$

2. For the following functions find the critical points, determine whether each critical point is a local minimum, local maximum or neither and provide the global minimum and global maximum on the specified interval.

	Function	**Interval**
1	$f(x) = x^3 - 4x^2 + 6x + 2$	$(-\infty, \infty)$
2	$g(x) = \frac{x^2+3x}{x+1}$	$(-3, 5)$
3	$h(x) = e^x - \ln(x+1)$	$(0, \infty)$
4	$p(x) = \frac{2x^3-x^2-4x+1}{x^2+1}$	$(-\infty, \infty)$
5	$q(x) = \sqrt{x^2 + 4x + 5}$	$(-2, 3)$
6	$r(x) = \sin(x) + \cos(x) + 2$	$(0, \pi)$
7	$s(x) = 3e^x - 2\ln(x)$	$(1, 4)$
8	$t(x) = \frac{x^3-2x^2-5x+1}{x+2}$	$(-5, 5)$
9	$u(x) = \sqrt[3]{x^2 + 1}$	$(-1, 2)$
10	$v(x) = \lvert x^2 - 6x + 8\rvert$	$(-\infty, \infty)$
11	$w(x) = \frac{\cos(x)}{1+\sin(x)}$	$(0, \pi)$
12	$z(x) = e^{-x} - \frac{\ln(x)}{2}$	$(0, 3)$
13	$b(x) = \frac{x^4-2x^3-5x^2+1}{x+2}$	$(-4, 4)$
14	$c(x) = \sqrt[4]{x^2 + 1}$	$(-1, 3)$

3. In the development of a new drug, a certain medication concentration in the bloodstream, $C(t)$, measured in miligrams per mililiter, is modeled by the function

$$C(t) = \frac{50}{t^2 - 4t + 8}$$

where t is the time in hours since the drug was administered. How much time does it take for the medication concentration to reach its maximum value? What is this value?

4. In a process, bacteria are utilized to produce a specific enzyme through fermentation. The rate of enzyme production, $E(t)$, measured in units per hour, is modeled by the function:

$$E(t) = 100t - 2t^2$$

where t is the time in hours since the beginning of the fermentation process. Determine the time at which the enzyme production is maximized during the process and find the maximum enzyme production rate.

5. Sketch the following functions. Go through the process of analyzing the domain, localize the critical points, find the intervals where the functions are increasing or decreasing, and determine the concavity of each function.

$f(x) = x^3 - 3x^2 - 9x + 5$	$g(x) = \frac{x^2}{x+1}$
$h(x) = \sqrt{x} + \frac{1}{x}$	$k(x) = e^{-x} \cdot \sin(x)$
$m(x) = \frac{x^2-4}{x-2}$	$r(x) = \frac{2x^2-5x-3}{x+1}$