Mathematics in Life Sciences **Winter Semester 2024**

List of Exercises 4

1. Solve the following exercises about continuity. Do not forget to check the right limit as well as the left limit when necessary.

   (a) Determine if the function $f(x) = 2x^2 - 3x + 1$ is continuous at $x = 2$.

   (b) Investigate the continuity of the function $g(x) = \frac{x^2-4}{x-2}$ at $x = 2$.

   (c) Check the continuity of the function $h(x) = \sqrt{x+3}$ at $x = -3$.

   (d) Investigate the continuity of the function $f(x) = \begin{cases} x^2, & \text{if } x \leq 0 \\ 1, & \text{if } 0 < x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$ at $x = 1$.

   (e) Check the continuity of the function $g(x) = \begin{cases} \sin(x), & \text{if } x < 0 \\ x, & \text{if } 0 \leq x < \pi \\ 2 - x & \text{if } x \geq \pi \end{cases}$ at $x = \pi$.

   (f) Determine if the function $h(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$.

   (g) Check the continuity of the function $j(x) = \begin{cases} x^3, & \text{if } x < 2 \\ 4x - 5, & \text{if } x \geq 2 \end{cases}$ at $x = 2$.

   (h) Investigate the continuity of the function $f(x) = \begin{cases} x \sin x, & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ at $x = 0$.

2. If possible, approximate a root of the following functions up to 1 decimal place in the given intervals. (For this exercise, you are allowed to use a calculator)

Function	Interval
$f(x) = x^3 - 2x - 5$	$[0, 4]$
$g(x) = \sin(x) - x + 1$	$[0, 2\pi]$
$h(x) = x^5 - 3x + 1$	$[0, 1]$
$j(x) = x^4 + x + 12$	$[0, 30]$
$k(x) = x^4 - x^3 - 2x^2 + x + 1$	$(1, 2)$
$m(x) = \cos(x) - x$	$[0, 1]$
$n(x) = x^5 - 5x^3 + 4x - 1$	$[1, 2]$
$p(x) = \tan(x) - x - 1$	$[0, \frac{\pi}{2})$
$q(x) = x^3 - 4x^2 + 3x + 2$	$[-2, 1]$
$r(x) = \sin(x) + \cos(x)$	$[0, \pi]$

3. Let $f(x) = \dfrac{x^2 - 2x - 8}{\sqrt{x} - 2}$.
   - What is the domain of f?
   - Is f continuous at 4?
   - Can we define $f(4)$ to make f continuous at 4? If so, what is the value?

4. Find all the points where $x^5 - x^4 - 5x^3 + x^2 + 8x + 4 > 0$. (Hint: This polynomial has exactly two solutions, namely 2 and -1)

5. Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \notin \mathbb{Q}. \end{cases}$$

   Find all the points where f is continuous.
   Hint: Recall that each point has arbitrarily close rational and irrational numbers.