Mathematics in Life Sciences **Winter Semester 2024**

List of Exercises 3

1. Find the limit of the following functions whenever it is possible.

(a) $\lim_{x\to -3} \frac{x^2-9}{x+3}$	(b) $\lim_{x\to 1} \frac{\sqrt{x}-x^2+1}{x-1}$	(c) $\lim_{x\to 1} \frac{x^2-1}{\lvert x-1\rvert}$
(d) $\lim_{x\to 2} \frac{x^2-4x+3}{x^2-5x+6}$	(e) $\lim_{x\to 0} \frac{\tan(x)}{x}$	(f) $\lim_{x\to \infty} \frac{2x^2-3x+1}{3x^2+2x-5}$
(g) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$	(h) $\lim_{x\to \pi/2} \tan(x)$	(i) $\lim_{x\to 0} \frac{\lvert 2x^2-5x+3\rvert}{x-1}$
(j) $\lim_{x\to -2} \frac{x^3+8}{x^2+2x-8}$	(k) $\lim_{x\to 4^+} \frac{x-4}{\sqrt{x-4}}$	(l) $\lim_{x\to \infty} \frac{3x^3-2x^2+5}{4x^3+x^2-6}$
(m) $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$	(n) $\lim_{x\to 2} \frac{x^2-4x+4}{x^2-3x-10}$	(o) $\lim_{x\to -1} \frac{x^2-3x+2}{x^2-2x-3}$

2. Calculate the following limits.

(a) $\lim_{x\to \infty} \frac{\lvert x+3\rvert-\lvert x-2\rvert}{x+1}$	(b) $\lim_{x\to -\infty} \frac{\lvert 2x-5\rvert-\lvert x+1\rvert}{3x-2}$
(c) $\lim_{x\to \infty} \frac{3x^2-2x+1}{2x^2+5x+7}$	(d) $\lim_{x\to -\infty} \frac{4x^3+2x^2-5}{2x^3-3x+1}$
(e) $\lim_{x\to \infty} \frac{\sqrt{4x^2+3x}-2x}{3x+5}$	(f) $\lim_{x\to -\infty} \frac{5x^2+3x-2}{3x^2-2x+1}$
(g) $\lim_{x\to \infty} \frac{\sin(2x)}{x}$	(h) $\lim_{x\to -\infty} \frac{x^2-1}{x^3+x+2}$

3. Consider the function $k(x) = x^2 \lvert \sin\left(\frac{1}{x}\right) \rvert$. Show that $0 \leq k(x) \leq x^2$ for all $x \in \mathbb{R}$. Find $\lim_{x\to 0} k(x)$ using the Squeeze Theorem.

4. Imagine you are driving a car on a highway, and your distance s (in meters) from a certain point is given by the function $s(t) = 30t - 2t^2$, where t is the time in seconds since you passed that point.

   (a) Determine the time at which you pass the point for the second time.

   (b) Calculate your average speed over the first 10 seconds after you pass the point for the first time.

   (c) Find the limit of your average speed as t approaches 0. Interpret the meaning of this limit in the context of the problem.

   Hint: The average speed over an interval $[a, b]$ *is given by* $\frac{s(b)-s(a)}{b-a}$.

5. Consider the function $f(x)$ defined as follows:

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Prove that $-1 \leq f(x) \leq 1$ for all x.

(b) Determine the value of $\lim_{x \to 0} f(x)$ if it exists. If it does not exist, explain why.