

List 1. Math for Electronics.

Exercise 1. A production line produces three components. For each component, record whether it is working (W) or failed (F) after testing.

- a) What is the sample space?
- b) Write down a list of the following events:
 - A : at least one component fails,
 - B : exactly two components work,
 - C : the first component works, and the third fails.
- c) Which outcomes belong to $A \cap B$? Which ones to $A \cup B$?
- d) Find A^c .

Exercise 2. A digital message consists of four bits. During transmission, each bit is either received correctly (C) or incorrectly (E).

- a) What is the sample space?
- b) How many outcomes are there in the sample space?
- c) Write down the event A that at least one bit is received incorrectly.
- d) Write down the event B that the first and last bits are received correctly.
- e) Describe the event A^c in words and list its outcomes.
- f) Write down an event that is neither a subset of A nor a subset of A^c .

Exercise 3. A temperature sensor measures a temperature between 0°C and 100°C .

- a) Suppose the sensor reports the temperature to the nearest degree. What is a suitable sample space?
- b) Suppose instead that the sensor reports the temperature to the nearest 0.1°C . What is a suitable sample space?
- c) Suppose the sensor can report any real-valued temperature between 0°C and 100°C . What is a suitable sample space?
- d) For the third case, define the event

$$A = \{T : 20 \leq T \leq 50\}.$$

Describe A^c in words.

Exercise 4. A system consists of two components, A and B . Each component can be either working (W) or failed (F).

- a) What is the sample space?
- b) Suppose the components are connected in series, so that the system works if and only if both components work. Write down the event A that the system is working.
- c) Write down the event B that the system has failed.
- d) Now suppose the components are connected in parallel, so that the system works if at least one component works. Write down the event A that the system is working.

Exercise 6. Design your own probability experiment.

- a) Construct an experiment whose sample space contains exactly eight outcomes.
- b) Write down the sample space.
- c) Define three different events.

- d) Give two events that are mutually exclusive.
- e) Give two events whose intersection is nonempty.
- f) Choose one of your events and write down its complement.

Exercise 6. A drone has 8 possible checkpoints, labeled from A through H. During a test flight, it must visit 5 different checkpoints, and it cannot visit the same checkpoint twice. How many different flights plans are possible?

Exercise 7. A robot can perform the following actions: Move forward, move backward, rotate left, rotate right, scan, jump, or briefly blink a led light. A program consists of 5 different actions.

- a) How many different programs can be created?
- b) How many different programs can be created with no action repeated?

Exercise 8. A certain foreign language has 50 different letters.

- a) How many 8 letter words are there?
- b) How many 12 letter words are there, where no letters are allowed to repeat.

Exercise 9. An engineer has 10 possible experimental procedures. A protocol consists of 5 different procedures, performed one after another. One particular procedure, the procedure P , is mandatory.

- a) How many protocols are possible?
- b) How many protocols have P as the first procedure?
- c) How many protocols have P as the last procedure?
- d) How many protocols have P somewhere in the middle?

Exercise 10. A circuit board has 12 distinct components, of which 5 are resistors, 4 are capacitors, and 3 are sensors.

- a) In how many ways can you select 4 components for testing?
- b) In how many ways can you select exactly 2 resistors and 2 capacitors?

Exercise 11. A project has 8 mechanical engineers and 6 electrical engineers. A team of 5 must be selected.

- a) How many possible teams are there?
- b) How many teams contain exactly 3 mechanical engineers?
- c) How many teams contain at least 2 electrical engineers?

Suppose one of the engineers is designated team leader. How many team + leader assignments are there?

Exercise 12. A device generates an ID consisting of 3 uppercase of the following letters: A,B,C,D,E,F,G, followed by 4 digits. Assume repetition is allowed.

- a) How many IDs are possible?
- b) How many are possible if no letter can be repeated?
- c) How many are possible if the four digits must all be different?
- d) How many IDs contain at least one zero?

Exercise 13. Suppose A, B are two events such that

$$P(A) = .6 \quad P(B) = .5$$

- Is it possible to have $P(A \cap B) = .3$? If yes, construct an example. That is, give a sample space and two events A and B for which these probabilities hold. If no, give a reason why this is not possible.
- Is it possible to have $P(A \cap B) = 0.05$?
- What is the smallest possible value for $P(A \cap B)$?

Exercise 14. Suppose that three events A, B and C satisfy the following:

$$\begin{aligned} P(A) &= .4 & P(B) &= .7 & P(C) &= .5 \\ P(A \cap B) &= .3 & P(A \cap C) &= .2 & P(B \cap C) &= .4 \\ P(A \cap B \cap C) &= .1 \end{aligned}$$

- Draw a Venn Diagram of these three events.
- Use that picture to calculate $P(A \cap B \cap C^c)$.
- Calculate the probability of each of the eight regions in the Venn Diagram.
- Is it possible for these numbers to describe probabilities? Explain.

Exercise 15. A box contains 10 electronic components, of which 3 are defective. Four components are selected at random, all of them at the same time. Find the probability that:

- none of them are defective,
- exactly one of them is defective,
- at least two are defective.

Exercise 16. A point is selected uniformly at random from a square of side 2. Inside the square is a circle of radius 1, centered at the square's center. What is the probability that the point falls inside the circle.

Exercise 17. A 4-digit PIN is chosen uniformly from all possible 4-digit PINs. What is the probability that

- all four digits are all different?
- exactly two digits are the same?
- at least two digits are the same?
- the PIN contains no zeroes?

Exercise 18. Suppose there are m people in a room, and assume that all 365 days are equally likely birthdays. Find the probability that at least two people share a birthday when

- $n = 2$,
- $n = 3$,
- $n = 10$,
- $n = 23$,
- $n = 50$.

Exercise 19. Consider a standard Poker deck of 52 playing cards. Five cards are dealt at random, and consider all 5-card hands are equally likely.

A hand can have one of the following types:

- One pair: two cards of the same rank, together with three cards of different ranks.
- Two pairs: two cards of one rank, two cards of another rank, and one card of a third rank.
- Three of a kind: three cards of the same rank, together with two cards of different ranks.
- Full house: three cards of one rank and two cards of another rank.
- Four of a kind: four cards of the same rank, together with one other card.
- Straight: five cards of consecutive ranks, but not all of the same suit.

- Flush: five cards of the same suit, but not a straight flush.
- Straight flush: five consecutive cards of the same suit, but not a royal flush.
- Royal flush: 10, J , Q , K , A , all of the same suit.
- High card: a hand that does not fall into any of the categories above.

For a straight, the ace may be either the lowest card $A, 2, 3, 4, 5$ or the highest card $10, J, Q, K, A$.

- Calculate the probability of obtaining each type of hand.
- Rank those hand types from most likely to least likely.

Exercise 20. One card is selected randomly from a standard 52-card deck.

- What is the probability that the card is a king given that it is a face card (J, Q or K)?
- What is the probability that the card is a face card given that it is a king?

Conclude that $P(A|B)$ may be different from $P(B|A)$.

Exercise 21. In a group of 100 students, 60 are taking programming, 40 are taking statistics and 25 are taking both. If a student is selected at random, calculate:

- $P(\text{statistics}|\text{programming})$,
- $P(\text{programming}|\text{statistics})$,
- $P(\text{statistics}|\text{not programming})$.

Exercise 21. A factory produces resistors on two production lines:

- Line A produces 60% of the resistors and has a defect rate of 2%,
- line B produces 40% of the resistors and has a defect rate of 5%.

A resistor is selected at random.

- What is the probability that it is defective?
- Given that the resistor is defective, what is the probability that it came from Line A?
- Are the events "Comes from line A" and "is defective" independent? Explain.

Exercise 22. A certain disease occurs in 2 out of every 100 people in a population. A medical test is used to detect the disease. The test has the following properties:

- If a person has the disease, the probability that the test gives a positive result is 0.95.
- If a person does not have the disease, the probability that the test gives a positive result is 0.10.

A person is selected at random from the population and takes the test.

- What is the probability that the person has the disease?
- What is the probability that the test gives a positive result?
- Given that the test gives a positive result, what is the probability that the person actually has the disease?

Exercise 23. Two six-sided dice are rolled. Let A be the event coding that the first dice shows an even number, and let B be the event that says that the second dice shows an even number. Are A and B independent? Calculate $P(A)$, $P(B)$ and $P(A \cap B)$ to justify your answer.

Exercise 24. A six-sided dice is rolled. Let A be the event saying that the result is even, and let B be the event that says that the result is greater than 3. Determine whether A and B are independent.

Exercise 25. An experiment produces the following results:

	B	B^c	Total
A	120	80	200
A^c	180	620	800
Total	300	700	1000

Determine whether A and B are independent.